

Mathematician Derrick Lehmer is known mostly as a number theorist, for instance, for the Lucas-Lehmer primality test for Mersenne numbers. But he also did work in the area of combinatorics, the art of enumerating and counting.

In 1965, he formulated a conjecture about the enumeration of all permutations of a sequence of objects, where every two subsequent permutations in that enumeration can be obtained from each other by swapping two adjacent objects.

This is also known as a minimal-change enumeration; these have numerous applications.

Consider three objects A, A, and B. In that case, we have three permutations: AAB, ABA, and BAA.

This is indeed a minimal-change enumeration.

For instance, you can obtain ABA from AAB by swapping the rightmost neighbor pair.

In contrast, AAB and BAA cannot appear one after the other in the enumeration.

In such an enumeration, each permutation appears — preferably — exactly once.

With AAB that is possible, but with AABB not (try it).

Lehmer conjectured that it will be possible when you allow a very limited type of duplications, viz. where the same neighbor pair is swapped twice in a row; thus, first \_\_AB\_\_, then \_\_BA\_\_, and finally \_\_AB\_\_ again. The third duplicates the first.

Lehmer called such a duplicate a 'spur'.

Fifty years later, in 2015, Tom Verhoeff proved Lehmer's conjecture for binary sequences, that is, sequences with only two kinds of objects.

For more than two kinds, the situation is more complex.